EXTRACTA MATHEMATICAE
Vol. **38**, Num. 2 (2023), 125 – 137

doi:10.17398/2605-5686.38.2.125
Available online December 8, 2023

Results on Lie ideals of prime rings with homoderivations

A. Sarikaya , Ö. Gölbaşi [@]

Department of Mathematics, Faculty of Science, Sivas Cumhuriyet University
Sivas, Turkey

eraysen345@gmail.com , ogolbasi@cumhuriyet.edu.tr

Received May 26, 2023
Accepted November 14, 2023

Presented by C. Martínez

Abstract: Let R be a prime ring of characteristic not 2 and U be a noncentral square closed Lie ideal of R. An additive mapping H on R is called a homoderivation if $H(xy) = H(x)H(y)+H(x)y+xH(y)$ for all $x, y \in R$. In this paper we investigate homoderivations satisfying certain differential identities on square closed Lie ideals of prime rings.

Key words: prime ring, Lie ideal, homoderivation, commutativity.

MSC (2020): 16W25, 16N60.

1. Introduction

Let R be an associative ring with center Z. For any $x, y \in R$ the symbol $[x, y]$ represents the Lie commutator $xy - yx$ and the Jordan product $xoy = xy+yx$. An additive subgroup U of R is said to be a Lie ideal of R if $[u, r] \in U$, for all $u \in U$, $r \in R$ and U is called a square closed Lie ideal of R if U is a Lie ideal and $u^2 \in U$ for all $u \in U$. If $u^2 \in U$ for all $u \in U$, then we have $(u + v)^2 \in U$ and so $(u + v)^2 - u^2 - v^2 = uv + vu \in U$, for all $u, v \in U$. Also $uv - vu \in U$, for all $u, v \in U$. Hence we find $2uv \in U$, for all $u, v \in U$.

Recall that a ring R is prime if for $x, y \in R$, $xRy = 0$ implies either $x = 0$ or $y = 0$. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. E.C. Posner studied conditions about derivations of a ring that imply commutativity of the ring in [8]. Over the last several years, a number of authors studied commutativity theorems for prime rings admitting automorphisms or derivations on appropriate subsets of R.

In 2000, El Sofy [5] defined a homoderivation on R as an additive mapping $H : R \to R$ satisfying $H(xy) = H(x)H(y) + H(x)y + xH(y)$ for all $x, y \in R$. An example of such mapping is to let $H(x) = f(x) - x$, for all $x, y \in R$ where

[@] Corresponding author

ISSN: 0213-8743 (print), 2605-5686 (online)

f is an endomorphism on R. It is clear that a homoderivation H is also a derivation if $H(x)H(y) = 0$ for all $x, y \in R$. In this case, $H(x)RH(y) = 0$ for all $x, y \in R$. Hence if R is a prime ring, then the only additive mapping which is both a derivation and a homoderivation is the zero mapping.

If $S \subseteq R$, then a mapping $f : R \to R$ preserves S if $f(S) \subseteq S$. A mapping $f : R \to R$ is zero-power valued on S if f preserves S and if for each $x \in S$, there exists a positive integer $n(x) > 1$ such that $f^{n(x)}(x) = 0$.

In [3], Daif and Bell proved that if R is semiprime ring, U is a nonzero ideal of R and d is a derivation of R such that $d([x, y]) = \pm[x, y]$, for all $x, y \in U$, then R contains a nonzero central ideal. There is also a growing literature on strong commutativity preserving (SCP) maps and derivations of operator algebras, prime rings and semiprime rings. In [7], this condition was examined by Melaibari et al. for homoderivations. On the other hand, Ashraf and Rehman showed that if R is a prime ring with a nonzero ideal U of R and d is a derivation of R such that $d(xy) \pm xy \in Z$, $d(xy) \pm yx \in Z$, for all $x, y \in U$, then R is commutative in [2]. Alharfie and Muthana proved similar results regarding homoderivations in [1].

The purpose of this paper is to study homoderivations satisfying certain differential identities on Lie ideals of prime rings. We will give new results in this field and generalize some theorems found in the literature.

2. Results

Throughout this paper, R will be a prime ring of characteristic not 2, U a square closed Lie ideal of R and H a homoderivation of R. For any $x, y \in R$, we make extensive use of basic commutator identities:

$$\begin{aligned}
[x, yz] &= y[x, z] + [x, y]z, \\
[xy, z] &= [x, z]y + x[y, z], \\
xo(yz) &= (xoy)z - y[x, z] = y(xoz) + [x, y]z, \\
(xy)oz &= x(yoz) - [x, z]y = (xoz)y + x[y, z].
\end{aligned}$$

Remark 1. For all $x, y \in R$, we get

$$\begin{aligned}
H([x, y]) &= H(xy - yx) = H(xy) - H(yx) \\
&= H(x)H(y) + H(x)y + xH(y) - H(y)H(x) - H(y)x - yH(x) \\
&= [H(x), H(y)] + [H(x), y] + [x, H(y)].
\end{aligned}$$

Lemma 1. ([4, Lemma 2]) *Let R be a prime ring of characteristic not 2 and U be a noncentral Lie ideal of R. Then $C_R(U) = Z$.*

Lemma 2. ([4, Lemma 4]) *Let R be a prime ring of characteristic not 2 and U be a noncentral Lie ideal of $R, a, b \in R$. If $aUb = (0)$ then $a = 0$ or $b = 0$.*

Lemma 3. *Let R be a prime ring of characteristic not 2 and U be a noncentral Lie ideal of $R, a \in R$. If $Ua = (0)$ $(aU = (0))$, then $a = 0$.*

Proof. Take an arbitrary nonzero b in R. $Ua = (0)$ implies $bUa = (0)$. Then $a = 0$ immediatly follows from Lemma 2. ∎

Lemma 4. ([6, Lemma 9]) *Let R be a prime ring of characteristic not 2 and U be a noncentral Lie ideal of $R, a \in R$. If $aoU = (0)$, then $a \in Z$.*

Lemma 5. ([6, Theorem 2]) *Let R be a prime ring of characteristic not 2, U a noncentral Lie ideal of R and H a homoderivation of R. If $H(U) \subseteq Z$, then $H = 0$.*

Theorem 1. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R and H a homoderivation which is zero-power valued on U. If $[H(u), H(v)] = [u, v]$ for all $u, v \in U$, then $H = 0$.*

Proof. By the hypothesis, we get

$$[H(u), H(v)] = [u, v] \qquad \text{for all } u, v \in U.$$

Replacing v by $2vw$ in the last equation, we have

$$\begin{aligned} 2[H(u), H(v)]H(w) + 2H(v)[H(u), H(w)] & \\ + 2[H(u), H(v)]w + 2H(v)[H(u), w] & \\ + 2[H(u), v]H(w) + 2v[H(u), H(w)] &= 2[u, v]w + 2v[u, w]. \end{aligned}$$

Using the hypothesis and $\mathrm{char} R \neq 2$, we obtain that

$$[u, v]H(w) + H(v)[u, w] + H(v)[H(u), w] + [H(u), v]H(w) = 0.$$

That is,

$$[u + H(u), v]H(w) + H(v)[H(u) + u, w] = 0.$$

Since H is zero-power valued on U, there exists an integer $n = n(u) > 1$ such that $H^n(u) = 0$. Replacing u by $u - H(u) + H^2(u) + \cdots + (-1)^{n-1}H^{n-1}(u)$ in this equation, we find

$$[u, v]H(w) + H(v)[u, w] = 0.$$

Replacing w by u in the last equation, we get

$$[u, v]H(u) = 0 \qquad \text{for all } u, v \in U.$$

Taking $2vw$ instead of v in the last equation and using this, we have

$$2[u, v]wH(u) = 0 \qquad \text{for all } u, v \in U.$$

Since char $R \neq 2$, we get

$$[u, v]UH(u) = (0) \qquad \text{for all } u, v \in U.$$

Now Lemma 2 implies that either $H(u) = 0$ or u belongs to the centralizer of U in R for all $u \in U$. Hence we have $H(u) = 0$ or $u \in Z$ for any $u \in U$ by Lemma 1.

Let $L = \{u \in U : u \in Z\}$ and $K = \{u \in U : H(u) = 0\}$. Clearly each of L and K is additive subgroup of U such that $U = L \cup K$. But a group can not be union of two proper subgroups. Hence $L = U$ or $K = U$. In the first case, $U \subseteq Z$, which forces our hypothesis. In the last case, we get $H(U) = (0)$, and so $H(U) \subseteq Z$. By Lemma 5, we obtain the required result. ∎

Theorem 2. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R and H a homoderivation which is zero-power valued on U. If one of the following conditions is satisfied then $H = 0$.*

(i) $H(u)H(v) = [u, v]$, *for all* $u, v \in U$.

(ii) $H(u)H(v) = u \circ v$, *for all* $u, v \in U$.

Proof. (i) By the hypothesis, we get

$$H(u)H(v) = [u, v] \qquad \text{for all } u, v \in U.$$

Replacing u by $2uw$ in this equation, we have

$$2H(u)H(w)H(v) + 2H(u)wH(v) + 2uH(w)H(v) = 2[u, v]w + 2u[w, v].$$

Using the hypothesis, we see that

$$2[u,w]H(v) + 2H(u)wH(v) = 2\,[u,v]\,w.$$

Taking $v = u$ in the above equation and using $\operatorname{char} R \neq 2$, we find that

$$[u,w]H(u) + H(u)wH(u) = 0.$$

By the hypothesis, we have

$$H(u)H(w)H(u) + H(u)wH(u) = 0. \tag{2.1}$$

That is,

$$H(u)(H(w) + w)H(u) = 0.$$

Since H is zero-power valued on U, there exists an integer $n = n(w) > 1$ such that $H^n(w) = 0$. Replacing w by $w - H(w) + H^2(w) + \cdots + (-1)^{n-1}H^{n-1}(w)$ in this equation, we get

$$H(u)wH(u) = 0.$$

That is, $H(u)UH(u) = (0)$, for all $u \in U$. By Lemma 2, we get $H(u) = 0$, for all $u \in U$, and so we conclude that $H = 0$ by Lemma 5.

(ii) We get

$$H(u)H(v) = u \circ v \qquad \text{for all } u, v \in U.$$

Replacing u by $2uw, w \in U$ in this equation, we get

$$2H(u)H(w)H(v) + 2H(u)wH(v) + 2uH(w)H(v) = 2u(w \circ v) - 2[u,v]w.$$

Using the hypothesis, we obtain that

$$2H(u)H(w)H(v) + 2H(u)wH(v) = -2[u,v]w.$$

Replacing v by u in this equation and using $\operatorname{char} R \neq 2$, we arrive at

$$H(u)H(w)H(u) + H(u)wH(u) = 0.$$

The rest of the proof is the same as equation (2.1). This completes the proof. ∎

Theorem 3. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R and H a homoderivation which is zero-power valued on U. If $uH(v) \pm uv \in Z$, for all $u, v \in U$, then $H = 0$.*

Proof. By the hypothesis, we get

$$uH(v) + uv \in Z, \qquad \text{for all } u, v \in U.$$

Replacing u by $2vu$ in this equation, we have

$$2v(uH(v) + uv) \in Z, \qquad \text{for all } u, v \in U.$$

Since R is prime ring, $uH(v) + uv \in Z$ and $\operatorname{char} R \neq 2$, we find that

$$v \in Z \quad \text{or} \quad uH(v) + uv = 0.$$

Let $L = \{v \in U : v \in Z\}$ and $K = \{v \in U : uH(v) + uv = 0, \text{ for all } u \in U\}$. Clearly each of L and K is additive subgroup of U such that $U = L \cup K$. But a group can not be union of two proper subgroups. Hence $L = U$ or $K = U$. In the first case, $U \subseteq Z$, which forces our hypothesis. So, we must have

$$uH(v) + uv = 0, \qquad \text{for all } u, v \in U.$$

Hence

$$u(H(v) + v)H(u) = 0, \qquad \text{for all } u, v \in U.$$

Since H is zero-power valued on U, there exists an integer $n = n(v) > 1$ such that $H^n(v) = 0$. Replacing v by $v - H(v) + H^2(v) + \cdots + (-1)^{n-1}H^{n-1}(v)$ in this equation, we get

$$uvH(u) = 0, \qquad \text{for all } u, v \in U.$$

Lemma 2 yields that $u = 0$ or $H(u) = 0$ for each $u \in U$. If $u = 0$, then we have $H(u) = 0$. Hence we get $H(u) = 0$, for all $v \in U$ for any cases. By Lemma 5, we obtain the required result.

Using the same arguments, we obtain a similar result for the case $uH(v) - uv \in Z$, for all $u, v \in U$. ∎

THEOREM 4. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R and H a homoderivation which is zero-power valued on U. If $uH(v) \pm vu \in Z$, for all $u, v \in U$, then $H = 0$.*

Proof. By the hypothesis, we get

$$uH(v) + vu \in Z, \qquad \text{for all } u, v \in U.$$

Replacing u by $2vu$ in this equation, we have

$$2v(uH(v) + vu) \in Z, \qquad \text{for all } u, v \in U.$$

Since R is prime ring, $uH(v) + vu \in Z$ and $\operatorname{char} R \neq 2$, we find that

$$v \in Z \quad \text{or} \quad uH(v) + vu = 0.$$

If we apply Brauer's Trick as we had done above, then we have

$$uH(v) + vu = 0, \qquad \text{for all } u, v \in U.$$

Writing v by $2uv$ in this equation, we obtain that

$$\begin{aligned} 0 &= 2uH(u)H(v) + 2uH(u)v + 2u^2H(v) + 2uvu \\ &= 2uH(u)(H(v) + v) + 2u(uH(v) + vu) \end{aligned}$$

and so

$$uH(u)(H(v) + v) = 0, \qquad \text{for all } u, v \in U.$$

Since H is zero-power valued on U, there exists an integer $n = n(v) > 1$ such that $H^n(v) = 0$. Replacing v by $v - H(v) + H^2(v) + \cdots + (-1)^{n-1}H^{n-1}(v)$ in this equation, we get

$$uH(u)v = 0, \qquad \text{for all } u, v \in U.$$

Lemma 3 yields that $uH(u) = 0$ for all $u \in U$.

By our hypothesis, we get $uH(u) + u^2 \in Z$. Using $uH(u) = 0$, we get $u^2 \in Z$, for all $u \in U$. Hence we have

$$(u + v)^2 = u^2 + v^2 + uv + vu \in Z$$

and so

$$uv + vu \in Z, \qquad \text{for all } u, v \in U.$$

Subtracting this equation from our hypothesis, we arrive at

$$uH(v) + uv \in Z, \qquad \text{for all } u, v \in U.$$

Hence we get $H = 0$ by Theorem 3.

By the same tecniques, we get the required result for the case $uH(v) - uv \in Z$, for all $u, v \in U$. ∎

Theorem 5. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R and H a homoderivation which is zero-power valued on U. If $uH(v) \pm [u,v] \in Z$, for all $u, v \in U$, then $H = 0$.*

Proof. Let us assume

$$uH(v) \pm [u,v] \in Z, \qquad \text{for all } u, v \in U.$$

Replacing u by $2vu$ in the above equation and using this, we arrive at

$$2v(uH(v) \pm [u,v]) \in Z, \qquad \text{for all } u, v \in U.$$

Since R is prime ring, $\operatorname{char} R \neq 2$ and $uH(v) \pm [u,v] \in Z$, we get

$$v \in Z \quad \text{or} \quad uH(v) \pm [u,v] = 0.$$

Define $L = \{v \in U : v \in Z\}$ and $K = \{v \in U : uH(v) \pm [u,v] = 0, \text{ for all } u \in U\}$. Applying Brauer's Trick, we have $L = U$ or $K = U$. In the first case, $U \subseteq Z$, which forces our hypothesis. So, we must have

$$uH(v) \pm [u,v] = 0, \qquad \text{for all } u, v \in U.$$

Writing v by $2vu$ in this equation, we obtain that

$$\begin{aligned} 0 &= 2uH(v)H(u) + 2uH(v)u + 2uvH(u) \pm 2[u,v]u \\ &= 2u(H(v)+v)H(u) + 2(uH(v) \pm [u,v])u. \end{aligned}$$

Using $\operatorname{char} R \neq 2$ and $uH(v) \pm [u,v] = 0$, we have

$$u(H(v)+v)H(u) = 0, \qquad \text{for all } u, v \in U.$$

Since H is zero-power valued on U, there exists an integer $n = n(v) > 1$ such that $H^n(v) = 0$. Replacing v by $v - H(v) + H^2(v) + \cdots + (-1)^{n-1}H^{n-1}(v)$ in this equation, we get

$$uvH(u) = 0, \qquad \text{for all } u, v \in U.$$

Lemma 2 yields that $u = 0$ or $H(u) = 0$ for each $u \in U$. If $u = 0$, then $H(u) = 0$. Hence we get $H(u) = 0$ for all $u \in U$. By Lemma 5, we obtain the required result. ∎

Theorem 6. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R and H a homoderivation which is zero-power valued on U. If $H(v)u \pm [u,v] \in Z$, for all $u, v \in U$, then $H = 0$.*

Proof. By the hypothesis, we get

$$H(v)u \pm [u,v] \in Z, \qquad \text{for all } u,v \in U.$$

Replacing u by $2uv$ in the this equation and using this, we find that

$$2(H(v)u \pm [u,v])v \in Z, \qquad \text{for all } u,v \in U.$$

Since R is prime ring, $\operatorname{char} R \neq 2$ and $H(v)u \pm [u,v] \in Z$, we have

$$v \in Z \quad \text{or} \quad H(v)u \pm [u,v] = 0.$$

Again using Brauer's Trick as we had done above, we arrive at

$$H(v)u \pm [u,v] = 0, \qquad \text{for all } u,v \in U.$$

Substituing v by $2uv$ in this equation, we obtain that

$$\begin{aligned} 0 &= 2H(u)H(v)u + 2H(u)vu + 2uH(v)u \pm u[u,v] \\ &= 2H(u)(H(v)+v)u + 2u(H(v)u \pm [u,v]). \end{aligned}$$

Since $H(v)u \pm [u,v] = 0$ and $\operatorname{char} R \neq 2$, we have

$$2H(u)(H(v) \pm v)u = 0, \ \text{for all } u,v \in U.$$

Using H is zero-power valued on U, we get $H^n(v) = 0$, for an integer $n = n(v) > 1$. Replacing v by $v - H(v) + H^2(v) + \cdots + (-1)^{n-1}H^{n-1}(v)$ in the last equation, we have

$$H(u)vu = 0, \qquad \text{for all } u,v \in U.$$

Lemma 2 yields that $u = 0$ or $H(u) = 0$ for each $u \in U$. If $u = 0$, then $H(u) = 0$. Hence we take $H(u) = 0$ for all $u \in U$. We get the required result by Lemma 5. ∎

THEOREM 7. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R, H a homoderivation which is zero-power valued on U and $H(U) \subseteq U$. If $[H(u),v] \pm uv \in Z$, for all $u,v \in U$, then $H = 0$.*

Proof. Let us assume

$$[H(u),v] \pm uv \in Z, \qquad \text{for all } u,v \in U.$$

Replacing v by $2vH(u)$ in the above equation and using this, we get

$$2([H(u),v] \pm uv)H(u) \in Z, \qquad \text{for all } u,v \in U.$$

Since R is prime ring, $\operatorname{char} R \neq 2$ and $[H(u),v] \pm uv \in Z$, we have

$$H(u) \in Z \quad \text{or} \quad [H(u),v] \pm uv = 0.$$

Let $L = \{u \in U : H(u) \in Z\}$ and $K = \{u \in U : [H(u),v] \pm uv = 0$, for all $v \in U\}$. Clearly each of L and K is additive subgroup of U such that $U = L \cup K$. But a group can not be union of two proper subgroups. Hence $L = U$ or $K = U$.

Now, we assume that $K = U$. Then we have

$$[H(u),v] \pm uv = 0, \qquad \text{for all } u,v \in U.$$

Substituing v by $2uv$ in this equation, we obtain that

$$\begin{aligned} 0 &= 2[H(u),uv] \pm 2u^2 v \\ &= 2u([H(u),v] \pm uv) + 2[H(u),u]v \end{aligned}$$

and so

$$2[H(u),u]v = 0, \qquad \text{for all } u,v \in U.$$

Since $\operatorname{char} R \neq 2$, we have

$$[H(u),u]v = 0, \qquad \text{for all } u,v \in U.$$

Lemma 3 yields that

$$[H(u),u] = 0, \qquad \text{for all } u,v \in U. \tag{2.2}$$

On the other hand, we get $[H(u),u] \pm u^2 \in Z$, by our hypothesis. Using $[H(u),u] = 0$, for all $u,v \in U$, we have $u^2 \in Z$, for all $u \in U$. Hence we get

$$(u+v)^2 = u^2 + v^2 + uv + vu \in Z$$

and so

$$uv + vu \in Z, \qquad \text{for all } u,v \in U.$$

Replacing u by $2uv$ in the above equation and using this, we have

$$2(uv + vu)v \in Z, \qquad \text{for all } u,v \in U.$$

By the primeness of R, char$R \neq 2$ and $uv + vu \in Z$, we find that

$$v \in Z \quad \text{ or } \quad uv + vu = 0.$$

Define $A = \{v \in U : v \in Z\}$ and $B = \{v \in U : uv + vu = 0, \text{ for all } v \in U\}$. Applying Brauer's Trick, we have $A = U$ or $B = U$. In the first case, $U \subseteq Z$, which forces our hypothesis. So, we must have

$$uv + vu = 0, \qquad \text{for all } u, v \in U.$$

Hence we arrive at $UoU = 0$, and so $U \subseteq Z$ by Lemma 4. This contradicts $U \nsubseteq Z$. So, we must have $L = U$, and so $H(u) \in Z$, for all $u \in U$. By Lemma 5, we get $H = 0$. This completes the proof. ∎

Theorem 8. *Let R be a prime ring of characteristic not 2, U a noncentral square closed Lie ideal of R, H a homoderivation which is zero-power valued on U and $H(U) \subseteq U$. If $[H(u), v] \pm vu \in Z$, for all $u, v \in U$, then $H = 0$.*

Proof. Let us assume

$$[H(u), v] \pm vu \in Z, \qquad \text{for all } u, v \in U.$$

Replacing v by $2H(u)v$ in the above equation and using this, we get

$$2H(u)([H(u), v] \pm vu) \in Z, \qquad \text{for all } u, v \in U.$$

Since R is prime ring, char $R \neq 2$ and $[H(u), v] \pm uv \in Z$, we have

$$H(u) \in Z \quad \text{ or } \quad [H(u), v] \pm vu = 0.$$

Define $L = \{u \in U : H(u) \in Z\}$ and $K = \{u \in U : [H(u), v] \pm vu = 0, \text{ for all } v \in U\}$. Clearly each of L and K is additive subgroup of U such that $U = L \cup K$. But a group can not be union of two proper subgroups. Hence $L = U$ or $K = U$.

Now, we assume that $K = U$. Then we have

$$[H(u), v] \pm vu = 0, \qquad \text{for all } u, v \in U.$$

Replacing v by $2uv$ in this equation, we obtain that

$$\begin{aligned} 0 &= 2[H(u), uv] \pm 2u^2 v \\ &= 2u([H(u), v] \pm uv) + 2[H(u), u]v \end{aligned}$$

and so

$$[H(u), u]v = 0, \qquad \text{for all } u, v \in U.$$

Lemma 3 yields that

$$[H(u), u] = 0, \qquad \text{for all } u, v \in U.$$

Using the same arguments after the equation (2.2) in the proof of Theorem 7, we get the required result. ∎

Example 1. Suppose

$$R = \left\{ \begin{pmatrix} a & b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

and

$$U = \left\{ \begin{pmatrix} 0 & u & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : u \in \mathbb{R} \right\}$$

R is a prime ring of characteristic not 2 and U a noncentral Lie ideal of R. Let us define

$$H : R \longrightarrow U, \qquad H \begin{pmatrix} a & b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It is clear that H is a zero-power valued nonzero homoderivation on R. Some commutativity conditions given above are satisfied. However, R is not commutative and U is noncentral.

Conclusion. Let R be a ring. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. The study of derivations in prime rings was initiated by E.C. Posner in [8]. Over the last several years, a number of authors studied commutativity theorems for prime rings admitting automorphisms or derivations on appropriate subsets of R. In 2000, El Sofy [5] defined a homoderivation on R as an additive mapping $H : R \to R$ satisfying $H(xy) = H(x)H(y) + H(x)y + xH(y)$ for all $x, y \in R$. After this definition several results appeared where the authors proved commutativity results for the domain of these mappings. The goal of this paper is to prove the commutativity of Lie ideals of prime rings with homoderivations satisfying some algebraic conditions and collect the information about the commutative structure of these rings.

References

[1] E.F. Alharfie, N.M. Muthana, The commutativity of prime rings with homoderivations, *International Journal of Advanced and Applied Sciences* **5** (5) (2018), 79 – 81.

[2] M. Ashraf, N. Rehman, On derivations and commutativity in prime rings, *East-West J. Math.* **3** (1) (2001), 87 – 91.

[3] H.E. Bell, M.N. Daif, On commutativity and strong commutativity-preserving maps, *Canad. Math. Bull.* **37** (1994), 443 – 447.

[4] J. Bergen, I.N. Herstein, J.W. Kerr, Lie ideals and derivation of prime rings, *J. Algebra* **71** (1981), 259 – 267.

[5] M.M. El Sofy, "Rings with some kinds of mappings", M.Sc. Thesis, Cairo University, Branch of Fayoum, 2000.

[6] Ö. Gölbaşi, Homoderivations on Lie ideals of prime rings, to appear in *Int. J. Math. Stat.*

[7] A. Melaibari, N. Muthana, A. Al-Kenani, Homoderivations on Rings, *Gen. Math. Notes* **35** (1) (2016), 1 – 8.

[8] E.C. Posner, Derivations in prime rings, *Proc. Amer. Math. Soc.* **8** (1957), 1093 – 1100.