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Virtually $(r; r_1, \ldots, r_n; s)$-nuclear multilinear operators

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Abstract: In this paper, the space of virtually $(r; r_1, \ldots, r_n; s)$-nuclear multilinear operators between Banach spaces is introduced, some of its properties are described and its topological dual is characterized as a Banach space of multiple absolutely $(r'; r'_1, \ldots, r'_n; s')$-summing multilinear operators.

Key words: Multilinear operators, nuclear operators, summing operators.

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1. Introduction

The nuclear operators between Banach spaces appeared in [5] when the author studied an infinite dimensional extension of the Malgrange theorem on existence and approximation of solutions for convolution equations (see also [7]). The concept of nuclear multilinear operators was extended and studied in [8]. For other related results we mention [9] and [10]. Matos [9] studied virtually $(r; r_1, \ldots, r_n)$-nuclear n-linear operators from $X_1 \times \cdots \times X_n$ into Y, and proved that, if the spaces X_k^*'s $(k = 1, \ldots, n)$ have the λ_k-bounded approximation property; then for $r, r_1, \ldots, r_n \in [1, +\infty[$ the topological dual of the space of these operators, endowed with a natural linear topology, is isomorphic isometrically to the space of all absolutely $(r', r'_1, \ldots, r'_n)$-summing operators from $X_1^* \times \cdots \times X_n^*$ into Y^* with $\frac{1}{r} + \frac{1}{r'} = 1$ and $\frac{1}{r_k} + \frac{1}{r'_k} = 1$; for r, r_k and $s \in [1, +\infty]$, $k = 1, \ldots, n$.

In [3] Cerna established the definition of $(r; r_1, \ldots, r_n; s)$-nuclear multilinear operators, which are the natural generalization of the concept of (r, p, s)-nuclear linear operator introduced by Laprested [6] (see also [11]).

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Motivated by these ideas and developments, in this paper we introduce and study the virtually $(r; r_1, \ldots, r_n; s)$-nuclear n-linear operators and we will prove a relation between the topological dual of virtually $(r; r_1, \ldots, r_n; s)$-nuclear n-linear operators and the multiple $(r'; r'_1, \ldots, r'_n; s')$-summing operators [2]. As a consequence we get the same result between the topological dual of the space of $(r; r_1, \ldots, r_n; s)$-nuclear n-linear operators from $X_1 \times \cdots \times X_n$ into Y [4] and to the space of all absolutely $(r', r'_1, \ldots, r'_n, s')$-summing operators from $X_1^* \times \cdots \times X_n^*$ into Y^* [1], for r, r_k and $s \in [1, +\infty]$, $k = 1, \ldots, n$.

The definitions and notations used in this paper are, in general, standard. Let $n \in \mathbb{N}$. As usual, an element j from $\mathbb{N}^n$ will be represented by $(j_1, \ldots, j_n)$ with $j_k \in \mathbb{N}$ and $k = 1, \ldots, n$. We also consider the finite families $(y_j)_{j \in \mathbb{N}_m^n}$ of elements of a Banach space with $\mathbb{N}_m = \{1, \ldots, m\}$. If $n = 1$, we omit $\mathbb{N}^n$ in the preceding notations. Let $X_1, \ldots, X_n; Y$ be Banach spaces over $\mathbb{K}$ (either $\mathbb{C}$ or $\mathbb{R}$). The space of all continuous n-linear operators $T : X_1 \times \cdots \times X_n \rightarrow Y$ will be denoted by $\mathcal{L}(X_1, \ldots, X_n; Y)$. It becomes a Banach space with the natural norm

$$\|T\| = \sup_{\|x^k\| \leq 1,\ k=1,\ldots,n} \left\| T\left(x^1, \ldots, x^n\right) \right\|.$$

We recall that a n-linear mappings $T \in \mathcal{L}(X_1, \ldots, X_n; Y)$ is said to be of finite type if it has a finite representation of the form

$$T = \sum_{i=1}^{m} \lambda_i \varphi_i^1 \times \cdots \times \varphi_i^n b_i,$$

where $\lambda_i \in \mathbb{K}$, $\varphi_i^k \in X_k^*$, $k = 1, \ldots, n$, $b_i \in Y$, $i = 1, \ldots, m$. We denote by $\mathcal{L}_f(X_1, \ldots, X_n; Y)$ the vector subspace of $\mathcal{L}(X_1, \ldots, X_n; Y)$ of all n-linear mappings of finite type.

If $r \in]0, +\infty[$, we denote by $l_r(Y; \mathbb{N}^n)$ or $(l_r(\mathbb{N}^n);$ if $Y = \mathbb{K})$, the vector space of all families $(y_j)_{j \in \mathbb{N}^n}$ of elements of Y such that

$$\left\| (y_j)_{j \in \mathbb{N}^n} \right\|_r = \left(\sum_{j \in \mathbb{N}^n} \|y_j\|_Y^r \right)^{\frac{1}{r}} < \infty.$$

We observe that $\|\cdot\|_r$ is a norm (r-norm, if $r < 1$) on $l_r(Y; \mathbb{N}^n)$ and defines a complete metrizable linear topology on it. We denote by $l_\infty(Y; \mathbb{N}^n)$ (or $l_\infty(\mathbb{N}^n)$, if $Y = \mathbb{K}$) the Banach space of all bounded families $(y_j)_{j \in \mathbb{N}^n}$ of elements of Y, with the norm

$$\left\|(y_j)_{j\in\mathbb{N}^n}\right\|_\infty = \sup_{j\in\mathbb{N}^n}\|y_j\|.$$

The Banach subspace of all families $(y_j)_{j\in\mathbb{N}^n}$ such that

$$\lim_{j_k\to+\infty,\ k=1,\dots,n}\|y_j\| = 0$$

is denoted by $c_0(Y;\mathbb{N}^n)$ (or $c_0(\mathbb{N}^n)$, if $Y=\mathbb{K}$).

If $0<s\le\infty$, we will write $l_r^w(Y;\mathbb{N}^n)$ (or $l_r^w(\mathbb{N}^n)$, if $Y=\mathbb{K}$) for the vector space of all families $(y_j)_{j\in\mathbb{N}^n}$ of elements of Y such that

$$\left\|(y_j)_{j\in\mathbb{N}^n}\right\|_{w,s} := \sup_{\|\psi\|_{Y^*}\le 1}\left(\sum_{j\in\mathbb{N}^n}|\psi(y_j)|^s\right)^{\frac{1}{s}} = \sup_{\|\psi\|_{Y^*}\le 1}\left\|(\psi(y_j))_{j\in\mathbb{N}^n}\right\|_s < \infty,$$

where Y^* denotes the topological dual of Y.

It is well-known that for $1\le s<\infty$ and $(\varphi_j)_{j\in\mathbb{N}^n}\in l_s^w(Y^*;\mathbb{N}_m^n)$, we have

$$\left\|(\varphi_j)_{j\in\mathbb{N}^n}\right\|_{w,s} = \sup_{\phi\in B_{Y^{**}}}\left(\sum_{j\in\mathbb{N}^n}|\phi(\varphi_j)|^s\right)^{\frac{1}{s}} = \sup_{y\in B_Y}\left\|(\varphi_j(y))_{j\in\mathbb{N}^n}\right\|_s.$$

Let $0<r,\ 1<p,\ s\le\infty$ such that

$$\frac{1}{t} = \frac{1}{r}+\frac{1}{p}+\frac{1}{s}, \text{ with } t\in\left]0,1\right].$$

An operator $T\in\mathcal{L}(X;Y)$ is $(r;p;s)$-nuclear (see, e.g., [6, 11]) if it has a representation of the form

$$T=\sum_{i=1}^{\infty}\lambda_i x_i\otimes y_i, \tag{1}$$

with $(\lambda_i)_i\in l_r$, if $r<\infty$ (or $(\lambda_i)_i\in c_0$, if $r=+\infty$), $(x_i)_i\in l_p^w(X^*)$ and $(y_i)_i\in l_s^w(Y)$. The vector space of all such operators is denoted by $\mathcal{N}_{(r;p;s)}(X;Y)$ and it is a complete metrizable topological vector space under the t-norm

$$\mu_{(r;p;s)}(T) = \inf\left\{\|(\lambda_i)_i\|_r\,\|(x_i)_i\|_{w,p}\,\|(y_i)_i\|_{w,s}\right\}$$

where the infimum is taken over all representations of T as in (1).

The definition of the virtually $(r;r_1,\dots,r_n)$-nuclear operators below was first given in [9].

We consider $r\in\,]0,+\infty]$, $r_k\in[1,+\infty]$, such that $r\le r_k,\ k=1,\dots,n$ and

$$1\le\frac{1}{t_n}=\frac{1}{r}+\frac{1}{r_1'}+\dots+\frac{1}{r_n'}.$$

Definition 1.1. An operator $T\in\mathcal{L}(X_1,\dots,X_n;Y)$ is said to be virtually $(r;r_1,\dots,r_n)$-nuclear if there is a representation of the form

$$T=\sum_{j\in\mathbb{N}^n}\lambda_j\phi_{j_1}^1\times\dots\times\phi_{j_n}^n b_j \tag{2}$$

with $(\lambda_j)_{j\in\mathbb{N}^n}\in l_r(\mathbb{N}^n)$, if $r<\infty$ (or $(\lambda_j)_{j\in\mathbb{N}^n}\in c_0(\mathbb{N}^n)$, if $r=+\infty$), $\left(\phi_i^k\right)_{i=1}^{\infty}\in l_{r_k'}^w(X_k^*)$, for $k=1,\dots,n$ and $(b_j)_{j\in\mathbb{N}^n}\in l_\infty(Y;\mathbb{N}^n)$.

The vector space of these operators is denoted by $\mathcal{L}_{VN}^{(r;r_1,\dots,r_n)}(X_1,\dots,X_n;Y)$ and we consider on it the t_n-norm

$$\|T\|_{VN,(r;r_1,\dots,r_n)}=\inf\left\|(\lambda_j)_{j\in\mathbb{N}^n}\right\|_r\left\|(b_j)_{j\in\mathbb{N}^n}\right\|_\infty\prod_{k=1}^{n}\left\|\left(\phi_i^k\right)_{i=1}^{\infty}\right\|_{w,r_k'},$$

where the infimum is taken over all representations of T as in (2).

The notion of absolutely $(r;r_1,\dots,r_n;s)$-summing multilinear operators was introduced by the first author in [1].

Definition 1.2. For $0<r,\ r_1,\dots,r_n<\infty$ and $0<s\le\infty$ with $\frac{1}{r}\le\frac{1}{r_1}+\dots+\frac{1}{r_n}+\frac{1}{s}$, an n-linear operator $T\in\mathcal{L}(X_1,\dots,X_n;Y)$ is $(r;r_1,\dots,r_n;s)$-summing if there is a constant $C>0$ such that for any $x_1^k,\dots,x_m^k\in X_k$, $(1\le k\le n)$, and any $\varphi_1,\dots,\varphi_m\in Y^*$, we have

$$\left(\sum_{i=1}^{m}\left|\varphi_i\left(T\left(x_i^1,\dots,x_i^n\right)\right)\right|^r\right)^{\frac{1}{r}}\le C\prod_{k=1}^{n}\left\|\left(x_i^k\right)_{i=1}^{m}\right\|_{w,r_k}\left\|(\varphi_i)_{i=1}^{m}\right\|_{w,s}.$$

We denote the vector space of these operators by $\mathcal{L}_{as,(r;r_1,\dots,r_n;s)}(X_1,\dots,X_n;Y)$ and the smallest C satisfying the above inequality by $\pi_{(r;r_1,\dots,r_n;s)}^n(T)$ which defines a norm (r-norm if $r<1$) on $\mathcal{L}_{as,(r;r_1,\dots,r_n;s)}(X_1,\dots,X_n;Y)$.

The following multilinear generalization of $(r;r_1,\dots,r_n;s)$-summing operators was recently introduced by Bernardino et al. in [2].

Definition 1.3. Let $n\in\mathbb{N}$, r, s, $r_1,\dots,r_n\geq 1$ and $X_1,\dots,X_n,Y$ be Banach spaces. A continuous multilinear operator $T:X_1\times\cdots\times X_n\longrightarrow Y$ is multiple $(r;r_1,\dots,r_n;s)$-summing if there is a $C>0$ such that

$$\left(\sum_{j\in\mathbb{N}_m^n}\left|\varphi_j\left(T(x_{j_1}^1,\dots,x_{j_n}^n)\right)\right|^r\right)^{\frac{1}{r}}\leq C\left\|(\varphi_j)_{j\in\mathbb{N}_m^n}\right\|_{w,s}\prod_{k=1}^n\left\|\left(x_i^k\right)_{i=1}^m\right\|_{w,r_k}$$

where $\frac{1}{r}\leq\frac{1}{r_1}+\cdots+\frac{1}{r_n}+\frac{1}{s}$, $x_1^k,\dots,x_m^k\in X_k$, $k=1,\dots,n$ and $(\varphi_j)_{j\in\mathbb{N}_m^n}\in l_s^w\left(Y^*;\mathbb{N}_m^n\right)$.

We denote by $\mathcal{L}_{mas}^{(r;r_1,\dots,r_n;s)}\left(X_1,\dots,X_n;Y\right)$ the vector space of these operators. The smallest C satisfying the above inequality defines a norm (r-norm if $r<1$) on $\mathcal{L}_{mas}^{(r;r_1,\dots,r_n;s)}\left(X_1,\dots,X_n;Y\right)$; it is denoted by $\|T\|_{mas(r;r_1,\dots,r_n;s)}$.

Remark 1.4. By choosing $(s=\infty)$ in Definition 1.3, we obtain the definition of fully (or multiple) $(r;r_1,\dots,r_n)$-summing n-linear operators presented in [9].

We also need the definition of the $(r;r_1,\dots,r_n;s)$-nuclear n-linear operators. The ideal of $(r;r_1,\dots,r_n;s)$-nuclear operators was introduced by Cerna [3] (see also [4]).

Definition 1.5. For $0<r\leq\infty$, $1\leq s$, $r_1,\dots,r_n\leq\infty$, such that $1\leq\frac{1}{r}+\frac{1}{r_1'}+\cdots+\frac{1}{r_n'}+\frac{1}{s'}$, $T\in\mathcal{L}\left(X_1,\dots,X_n;Y\right)$ is called $(r;r_1,\dots,r_n;s)$-nuclear if it has the form

$$T=\sum_{i=1}^{+\infty}\lambda_i\phi_i^1\times\cdots\times\phi_i^n b_i,\tag{3}$$

with $(\lambda_i)_{i\in\mathbb{N}}\in l_r\left(\mathbb{N}\right)$, if $r<\infty$ (or $(\lambda_i)_{i\in\mathbb{N}}\in c_0\left(\mathbb{N}\right)$, if $r=+\infty$), $\left(\phi_i^k\right)_{i\in\mathbb{N}}\in l_{r_k'}^w\left(X_k^*\right)$ for $k=1,\dots,n$ and $(b_i)_{i\in\mathbb{N}}\in l_{s'}^w\left(Y\right)$. The set of $(r;r_1,\dots,r_n;s)$-nuclear operators satisfying the definition is a vector space and is denoted by $\mathcal{N}_{(r;r_1,\dots,r_n;s)}\left(X_1,\dots,X_n;Y\right)$. Considering that

$$N_{(r;r_1,\dots,r_n;s)}\left(T\right)=\inf\left\|(\lambda_i)_{i\in\mathbb{N}}\right\|_r\left\|(b_i)_{i\in\mathbb{N}}\right\|_{w,s'}\prod_{k=1}^n\left\|\left(\phi_i^k\right)_{i\in\mathbb{N}}\right\|_{w,r_k'},$$

where the infimum is taken over all possible representations of T described in (3), we obtain a t-norm with

$$\frac{1}{t}=\frac{1}{r}+\frac{1}{r_1'}+\cdots+\frac{1}{r_n'}+\frac{1}{s'}.$$

2. Virtually $(r;r_1,\ldots,r_n;s)$-nuclear n-linear operators

We consider $r\in\,]0,+\infty]$, $s,r_k\in[1,+\infty]$, $k=1,\ldots,n$, such that $1\le\frac{1}{t_n}=\frac{1}{r}+\frac{1}{r_1'}+\cdots+\frac{1}{r_n'}+\frac{1}{s'}$.

Definition 2.1. An operator $T\in\mathcal{L}(X_1,\ldots,X_n;Y)$ is said to be virtually $(r;r_1,\ldots,r_n;s)$-nuclear if there are $(\lambda_j)_{j\in\mathbb{N}^n}\in l_r(\mathbb{N}^n)$, if $r<\infty$ (or $(\lambda_j)_{j\in\mathbb{N}^n}\in c_0(\mathbb{N}^n)$, if $r=+\infty$), $\left(\phi_i^k\right)_{i=1}^{\infty}\in l_{r_k'}^w(X_k^*)$, for $k=1,\ldots,n$ and $(b_j)_{j\in\mathbb{N}^n}\in l_{s'}^w(Y;\mathbb{N}^n)$ such that

$$T=\sum_{j\in\mathbb{N}^n}\lambda_j\phi_{j_1}^1\times\cdots\times\phi_{j_n}^n b_j. \tag{4}$$

We denote the vector space of all such operators by $\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)$, with the t_n-norm

$$\|T\|_{VN,(r;r_1,\ldots,r_n;s)}=\inf\left\|(\lambda_j)_{j\in\mathbb{N}^n}\right\|_r\left\|(b_j)_{j\in\mathbb{N}^n}\right\|_{w,s'}\prod_{k=1}^{n}\left\|\left(\phi_i^k\right)_{i=1}^{\infty}\right\|_{w,r_k'},$$

where the infimum is taken over all representations of T as in (4). This t_n-normed space is a complete metrizable topological vector space.

Remarks 2.2. (a) By choosing $s'=\infty$ in Definition 2.1, we obtain virtually $(r;r_1,\ldots,r_n)$-nuclear n-linear operators presented in Definition 1.1.
(b) We have $\mathcal{N}_{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)\subset\mathcal{L}_{VN}^{(r;r_1,\ldots,r_m;s)}(X_1,\ldots,X_n;Y)$ and

$$\|T\|\le\|T\|_{VN,(r;r_1,\ldots,r_n;s)}\le N_{(r;r_1,\ldots,r_n;s)}(T),$$

for every T is in $\mathcal{N}_{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)$.

By definition every T in $\mathcal{L}_f(X_1,\ldots,X_n;Y)$ has a finite representation

$$T=\sum_{j\in\mathbb{N}_m^n}\lambda_j\phi_{j_1}^1\times\cdots\times\phi_{j_n}^n b_j. \tag{5}$$

It is clear that we have a t_n-norm on $\mathcal{L}_f(X_1,\ldots,X_n;Y)$ defined by

$$\|T\|_{VN_f,(r;r_1,\ldots,r_n;s)}=\inf\left\|(\lambda_j)_{j\in\mathbb{N}_m^n}\right\|_r\left\|(b_j)_{j\in\mathbb{N}_m^n}\right\|_{w,s'}\prod_{k=1}^{n}\left\|\left(\phi_i^k\right)_{i=1}^{m}\right\|_{w,r_k'},$$

where the infimum is taken over all finite representations of T as in (5).

The next result collects some elementary facts about virtually $(r;r_1,\dots,r_n;s)$-nuclear n-linear operators.

PROPOSITION 2.3. (i) *The vector space $\mathcal{L}_f(X_1,\dots,X_n;Y)$ of the continuous n-linear operators of finite type is dense in $\mathcal{L}_{VN}^{(r;r_1,\dots,r_n;s)}(X_1,\dots,X_n;Y)$.*

(ii) *Ideal property: If $E_1,\dots,E_n$, and Y_0 are Banach spaces and $T\in\mathcal{L}(X_1,\dots,X_n;Y)$, $S_k\in\mathcal{L}(E_k,X_k)$, $k=1,\dots,n$, and $R\in\mathcal{L}(Y,Y_0)$ with T virtually $(r;r_1,\dots,r_n;s)$-nuclear, then $R\circ T\circ(S_1,\dots,S_n)$ is virtually $(r;r_1,\dots,r_n;s)$-nuclear and*

$$\left\|R\circ T\circ(S_1,\dots,S_n)\right\|_{VN,(r;r_1,\dots,r_n;s)}\le\|R\|\,\|T\|_{VN,(r;r_1,\dots,r_n;s)}\prod_{k=1}^{n}\|S_k\|.$$

(iii) *$T\in\mathcal{L}(X_1,\dots,X_n;Y)$ is virtually $(r;r_1,\dots,r_n;s)$-nuclear if and only there are bounded linear operators $A_k\in\mathcal{L}(X_k;l_{r_k'})$, $k=1,\dots,n$, $B\in\mathcal{L}(l_1(\mathbb{N}^n);Y)$ and $(\lambda_j)_{j\in\mathbb{N}^n}\in l_r(\mathbb{N}^n)$, if $r<\infty$ (or $(\lambda_j)_{j\in\mathbb{N}^n}\in c_0(\mathbb{N}^n)$, if $r=+\infty$), such that*

$$T=B\circ\mathcal{D}_{(\lambda_j)_{j\in\mathbb{N}^n}}\circ(A_1,\dots,A_n),$$

where $\mathcal{D}_{(\lambda_j)_{j\in\mathbb{N}^n}}:l_{r_1'}\times\cdots\times l_{r_n'}\longrightarrow l_1(\mathbb{N}^n)$ defined by $\mathcal{D}_{(\lambda_j)_{j\in\mathbb{N}^n}}\big((\xi^1_{j_1})_{j_1=1}^{\infty},\dots,(\xi^n_{j_n})_{j_n=1}^{\infty}\big)=(\lambda_j\xi^1_{j_1}\cdots\xi^n_{j_n})_{j\in\mathbb{N}^n}$ for $(\xi^1_{j_1})_{j_1=1}^{\infty}\in l_{r_1'}$, is a virtually $(r;r_1,\dots,r_n;s)$-nuclear with

$$\left\|\mathcal{D}_{(\lambda_j)_{j\in\mathbb{N}^n}}\right\|_{VN,(r;r_1,\dots,r_n;s)}=\left\|(\lambda_j)_{j\in\mathbb{N}^n}\right\|_r.$$

In this case

$$\|T\|_{VN,(r;r_1,\dots,r_n;s)}=\inf\|B\|\left\|(\lambda_j)_{j\in\mathbb{N}^n}\right\|_r\prod_{k=1}^{n}\|A_k\|,$$

where the infimum is taken over all such factorizations.

3. DUALITY

The natural question is to find out when we have

$$\|T\|_{VN,(r;r_1,\dots,r_n;s)}=\|T\|_{VN_f,(r;r_1,\dots,r_n;s)},$$

for each $T\in\mathcal{L}_f(X_1,\dots,X_n;Y)$.

Of course we have

$$\|T\|_{VN,(r;r_1,\ldots,r_n;s)} \leq \|T\|_{VN_f,(r;r_1,\ldots,r_n;s)} .$$

Below we will see that the reverse implication holds to be true for some certain Banach spaces X_k's $(k=1,\ldots,n)$. We start with finite dimensional spaces X_k's. The following theorem can be proved as in [9, Proposition 4.6].

Theorem 3.1. *If the spaces X_k $(k=1,\ldots,n)$ are finite dimensional vector spaces, then*

$$\|T\|_{VN_f,(r;r_1,\ldots,r_n;s)} \leq \|T\|_{VN,(r;r_1,\ldots,r_n;s)} ,$$

for every $T \in \mathcal{L}_f(X_1,\ldots,X_n;Y)$.

As in [9, Proposition 4.8], we get the following, which extends Theorem 3.1 to infinite dimensional Banach spaces with the λ-bounded approximation property (λ-BAP, for short).

Proposition 3.2. *If the spaces X_k^*'s $(k=1,\ldots,n)$ have the λ_k-BAP, then*

$$\|T\|_{VN,(r;r_1,\ldots,r_n;s)} \geq \|T\|_{VN_f,(r;r_1,\ldots,r_n;s)} ,$$

for all $T \in \mathcal{L}_f(X_1,\ldots,X_n;Y)$.

Proof. We consider $T_k \in \mathcal{L}\big(X_k;\mathcal{L}(X_1,\ldots,X_{k-1},X_{k+1},\ldots,X_n;Y)\big)$, defined by

$$T_k\big(x^k\big)\big(x^1,\ldots,x^{k-1},x^{k+1},\ldots,x^n\big) = T\big(x^1,\ldots,x^{k-1},x^k,x^{k+1},\ldots,x^n\big),$$

for $x^k \in X_k,\ k=1,\ldots,n$.

Since X_k^* has the λ_k-bounded approximation property for some $\lambda_k > 0$, given $\epsilon > 0$, we can find $S_k \in \mathcal{L}_f(D_k,X_k)$, such that $T_k = T_k \circ S_k$ and $\|S_k\| \leq (1+\epsilon)\lambda_k$. Hence, for all $x^k \in X_k$, for $k=1,\ldots,n$, we have

$$T\big(x^1,\ldots,x^{k-1},S_k\big(x^k\big),x^{k+1},\ldots,x^n\big) = T\big(x^1,\ldots,x^{k-1},x^k,x^{k+1},\ldots,x^n\big).$$

Now, we can write

$$T\left(x^1,\ldots,x^n\right) = T\circ(S_1,\ldots,S_n)\left(x^1,\ldots,x^n\right), \quad \forall x^k \in X_k,\ k=1,\ldots,n.$$

If J_k denotes the natural injection from $S_k(D_k)$ into X_k, we can write $S_k = J_k \circ \widetilde{S}_k$ $\big(\widetilde{S}_k \in \mathcal{L}_f(D_k,S_k(D_k))\big)$, with $\big\|\widetilde{S}_k\big\| = \|S_k\|$. Therefore we can say that $T\circ(J_1,\dots,J_n)\in\mathcal{L}_f\left(\left(S_1\left(D_1\right),\dots,S_n\left(D_n\right)\right);Y\right)$. By Theorem 3.1 and Proposition 2.3 (ii) we have

$$\begin{aligned}\|T\|_{VN_f,(r;r_1,\dots,r_n;s)} &= \|T\circ(S_1,\dots,S_n)\|_{VN_f,(r;r_1,\dots,r_n;s)}\\ &\le \|T\|_{VN,(r;r_1,\dots,r_n;s)}\prod_{k=1}^{n}\|S_k\|\\ &\le \|T\|_{VN,(r;r_1,\dots,r_n;s)}(1+\epsilon)^n\prod_{k=1}^{n}\lambda_k.\end{aligned}$$

This implies that

$$\|T\|_{VN_f,(r;r_1,\dots,r_n;s)}\le\left(\prod_{k=1}^{n}\lambda_k\right)\|T\|_{VN,(r;r_1,\dots,r_n;s)}.$$

For each $\epsilon>0$, we choose a representation

$$T=\sum_{j\in\mathbb{N}^n}\sigma_j\phi_{j_1}^1\times\cdots\times\phi_{j_n}^n y_j$$

such that

$$\left\|(\sigma_j)_{j\in\mathbb{N}^n}\right\|_r\left\|(y_j)_{j\in\mathbb{N}^n}\right\|_{w,s'}\prod_{k=1}^{n}\left\|\left(\phi_i^k\right)_{i=1}^{\infty}\right\|_{w,r_k'}\le(1+\epsilon)\|T\|_{VN,(r;r_1,\dots,r_n;s)}.$$

We can find $m\in\mathbb{N}$ such that

$$\left(\prod_{k=1}^{n}\lambda_k\right)\left\|\sum_{j\in\mathbb{N}^n/\mathbb{N}_m^n}\sigma_j\phi_{j_1}^1\times\cdots\times\phi_{j_n}^n y_j\right\|_{VN_f,(r;r_1,\dots,r_n;s)}\le\epsilon\|T\|_{VN,(r;r_1,\dots,r_n;s)}.$$

We use the triangular inequality for t_n-norms in order to write

$$\begin{aligned}\left(\|T\|_{VN_f,(r;r_1,\dots,r_n;s)}\right)^{t_n} \le &\left(\left\|\sum_{j\in\mathbb{N}_m^n}\sigma_j\phi_{j_1}^1\times\cdots\times\phi_{j_n}^n y_j\right\|_{VN_f,(r;r_1,\dots,r_n;s)}\right)^{t_n}\\ &+\left(\left\|\sum_{j\in\mathbb{N}^n/\mathbb{N}_m^n}\sigma_j\phi_{j_1}^1\times\cdots\times\phi_{j_n}^n y_j\right\|_{VN_f,(r;r_1,\dots,r_n;s)}\right)^{t_n}\end{aligned}$$

$$\leq (1+\epsilon)^{t_n} \left(\|T\|_{VN,(r;r_1,\ldots,r_n;s)} \right)^{t_n}$$
$$+ \left(\prod_{k=1}^{n} \lambda_k \right)^{t_n} \left(\left\| \sum_{j \in \mathbb{N}^n / \mathbb{N}^n_m} \sigma_j \phi^1_{j_1} \times \cdots \times \phi^n_{j_n} y_j \right\|_{VN,(r;r_1,\ldots,r_n;s)} \right)^{t_n}$$
$$\leq \left[(1+\epsilon)^{t_n} + \epsilon^{t_n} \right] \left(\|T\|_{VN,(r;r_1,\ldots,r_n;s)} \right)^{t_n} .$$

Since $\epsilon > 0$ is arbitrary we have

$$\|T\|_{VN_f,(r;r_1,\ldots,r_n;s)} \leq \|T\|_{VN,(r;r_1,\ldots,r_n;s)} ,$$

and this proves the theorem. ∎

For Banach spaces with λ-bounded approximation property, Proposition 3.2 can be seen as a generalization of a result obtained by B. Cerna [4, Lemma 2.1].

Now, we also give another generalization of [4, Lemma 2.1].

Proposition 3.3. *Let $T : X_1 \times \cdots \times X_n \longrightarrow L_s(\Omega, \mu)$ be defined by*

$$T\left(x^1, \ldots, x^n\right) = \sum_{j \in \mathbb{N}^n_m} \lambda_j \phi^1_{j_1}\left(x^1\right) \cdots \phi^n_{j_n}\left(x^n\right) b_j ,$$

where $\frac{1}{s} = \frac{1}{r'_1} + \cdots + \frac{1}{r'_n}$. Then, $\|T\|_{VN_f,(\infty;r_1,\ldots,r_n;s)} = \|T\|_{VN,(\infty;r_1,\ldots,r_n;s)} = \|T\|$.

Proof. It is clear that for $\frac{1}{s} = \frac{1}{r'_1} + \cdots + \frac{1}{r'_n}$, we have

$$\|T\| \leq \|T\|_{VN,(\infty;r_1,\ldots,r_n;s)} \leq \|T\|_{VN_f,(\infty;r_1,\ldots,r_n;s)} .$$

Moreover,

$$\|T\| \left\|x^1\right\| \cdots \|x^n\| \geq \left(\int_\Omega \left| \sum_{j \in \mathbb{N}^n_m} \lambda_j \phi^1_{j_1}\left(x^1\right) \cdots \phi^n_{j_n}\left(x^n\right) b_j(t) \right|^s d\mu(t) \right)^{1/s} \qquad (6)$$

Since $\phi^i_{j_i}$ is surjective there exists $\xi_i \in X_i$ such that $\phi^i_{j_i}(\xi_i) = M_i / 2^{j_i / r'_i}$, where

$$M_i = \sup_{\|x^i\|_{X_i} \leq 1} \left(\sum_{j_i=1}^{m} \left| \left\langle \phi^i_{j_i}, x^i \right\rangle \right|^{r'_i} \right)^{1/r'_i} ,$$

We will show that $\|\xi_i\| \le 1$ and $M_i < +\infty$ for $i = 1,\dots,n$. From the definition of M_i for a fixed i and for $\epsilon > 0$ we have

$$M_i \|\xi_i\| < (1+\epsilon)\left(\sum_{j_i=1}^{m} M_i^{r'_i}/2^{j_i}\right)^{1/r'_i},$$

which implies that

$$\|\xi_i\| < (1+\epsilon)\,, \text{ for all } \epsilon > 0.$$

So, considering $\|\xi_i\| < 1$ in equation (6) we have

$$\|T\| \ge \left(\int_\Omega \left|\sum_{j\in\mathbb{N}_m^n} \lambda_j M_1/2^{j_1/r'_1}\cdots M_n/2^{j_n/r'_n} b_j(t)\right|^s d\mu(t)\right)^{1/s},$$

if $k = \max\{j_1,\dots,j_n\}$ we get

$$\|T\| \ge \left(\int_\Omega \left|\sum_{j\in\mathbb{N}_m^n} \lambda_j \frac{b_j(t)}{2^{k/s}}\right|^s d\mu(t)\right)^{1/s} \prod_{i=1}^{n} M_i. \tag{7}$$

Let $z(t) = \sum_{j\in\mathbb{N}_m^n} \lambda_j \frac{b_j(t)}{2^{k/s}}$, then for all $s \ge 1$ we have

$$|\langle\varphi, z\rangle| = \left|\sum_{j\in\mathbb{N}_m^n} \lambda_j \left\langle \varphi, \frac{b_j}{2^{k/s}}\right\rangle\right| \le \|\varphi\|\,\|z\|\,. \tag{8}$$

By renumbering multi-finite indices $j \in \mathbb{N}_m^n$, we can rewrite this finite sum as

$$z(t) = \sum_{k=1}^{f(m,n)} \frac{b_k}{2^{k/s}}\,.$$

In addition, let $M = span_{k\in\{1,\dots,f(m,n)\}-k_0}\left\{\frac{b_k}{2^{k/s}}\right\}$ where k_0 is a fixed number belongs to $\{1,\dots,f(m,n)\}$, and $f(m,n) \in \mathbb{N}$. Moreover, as a consequence of the Hahn-Banach theorem there exists φ such that $\|\varphi\| = \frac{1}{d}$, $\langle\varphi, x\rangle = 0$ for all $x \in M$ and $\left\langle\varphi, \frac{b_{k_0}}{2^{k_0/s}}\right\rangle = 1$, where $d = \inf_{x\in M}\left\|x - \frac{b_{k_0}}{2^{k_0/s}}\right\|$ and further one can choose λ_{k_0} such that

$$|\lambda_{k_0}| = \max_{k=1,\ldots,f(m,n)} |\lambda_k| = \left\| (\lambda_j)_{j\in\mathbb{N}^n} \right\|_\infty ; \text{ where } j = (j_1, \ldots, j_n) .$$

Taking into account these last relations in equation (8) we can get,

$$\|z\| \geq |\lambda_{k_0}| \, d. \tag{9}$$

Since $x = \sum_{k=1,\, k\neq k_0}^{f(m,n)} \frac{-b_k}{2^{k/s}} \in M$, then for a given $\epsilon > 0$, we have

$$(1+\epsilon)\, d > \left\| \sum_{k=1}^{f(m,n)} \frac{b_k}{2^{k/s}} \right\| .$$

Therefore, from (9) we get

$$(1+\epsilon)\, \|z\| > \left\| (\lambda_j)_{j\in\mathbb{N}^n} \right\|_\infty \left\| \sum_{k=1}^{f(m,n)} \frac{b_k}{2^{k/s}} \right\| . \tag{10}$$

We know that

$$\left\| (b_j)_{j\in\mathbb{N}^n} \right\|_{w,s'} = \sup_{\|\psi\|_s \leq 1} \left(\sum_{j\in\mathbb{N}_m^n} |\psi(b_j)|^{s'} \right)^{1/s'} = \sup_{a\in B_{l_s^{f(m,n)}}} \left\| \sum_{k=1}^{f(m,n)} a_k b_k \right\| ,$$

and since $a_k = \frac{1}{2^{k/s}}$ for $k = 1, \ldots, f(m,n)$, given $\widetilde{\epsilon} > 0$ we have

$$(1+\widetilde{\epsilon}) \left\| \sum_{k=1}^{f(m,n)} \frac{b_k}{2^{k/s}} \right\| \geq \left\| (b_j)_{j\in\mathbb{N}^n} \right\|_{w,s'} .$$

From the last relation and the equation (10) we obtain

$$(1+\epsilon)(1+\widetilde{\epsilon}) \|z\| > \left\| (\lambda_j)_{j\in\mathbb{N}^n} \right\|_\infty \left\| (b_j)_{j\in\mathbb{N}^n} \right\|_{w,s'} \text{ for all } \epsilon \text{ and } \widetilde{\epsilon} > 0. \tag{11}$$

Therefore, from the relations (7) and (11) we get

$$\begin{aligned} \|T\| &\geq \left\| (\lambda_j)_{j\in\mathbb{N}^n} \right\|_\infty \left\| (b_j)_{j\in\mathbb{N}^n} \right\|_{w,s'} \prod_{i=1}^{n} M_i \\ &\geq \|T\|_{VN_{f,(\infty;r_1,\ldots,r_n;s)}} . \end{aligned}$$

∎

We will prove a new link between the topological dual of virtually $(r;r_1,\ldots,r_n;s)$-nuclear n-linear operators and multiple $(r';r'_1,\ldots,r'_n;s')$-summing operators. The proof of the next theorem is similar to the proof of Theorem 7.3.1 in [10]. We included the detailed proof here for completeness.

THEOREM 3.4. *If the spaces X_k^*'s $(k=1,\ldots,n)$ have the λ_k- BAP, then the topological dual of $\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)$ is isomorphic isometrically to $\mathcal{L}_{mas}^{(r';r'_1,\ldots,r'_n;s')}(X_1^*,\ldots,X_n^*;Y^*)$, for $r,r_k\in[1,+\infty[$, $k=1,\ldots,n$ through the mapping $\mathcal{B}$ define by*

$$\mathcal{B}(\Psi)\left(\phi^1,\ldots,\phi^n\right)(b)=\Psi\left(\phi^1\times\cdots\times\phi^n b\right),$$

for all $b\in Y$, $\phi^k\in X_k^*,\ k=1,\ldots,n$ and $\Psi\in\left(\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)\right)^*$.

Proof. It is easy to see that the correspondence

$$\Psi\in\left(\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)\right)^*\longrightarrow\mathcal{B}(\Psi)\in\mathcal{L}_{mas}^{(r';r'_1,\ldots,r'_n;s')}(X_1^*,\ldots,X_n^*;Y^*)$$

defined by

$$\mathcal{B}(\Psi)\left(\phi^1,\ldots,\phi^n\right)(b)=\Psi\left(\phi^1\times\cdots\times\phi^n b\right),\ \phi^k\in X_k^*,\ k=1,\ldots,n \text{ and } b\in Y,$$

is linear and injective. To show the surjectivity let $T\in\mathcal{L}_{mas}^{(r';r'_1,\ldots,r'_n;s')}(X_1^*,\ldots,X_n^*;Y^*)$ and consider the linear functional Ψ_T on the space $\left(\mathcal{L}_f(X_1,\ldots,X_n;Y),\|\cdot\|_{VN_{f,(r;r_1,\ldots,r_n;s)}}\right)$ given by

$$\Psi_T(S)=\sum_{j\in\mathbb{N}_m^n}\lambda_j T\left(\varphi_{j_1}^1,\ldots,\varphi_{j_n}^n\right)(b_j)$$

for every $S\in\mathcal{L}_f(X_1,\ldots,X_n;Y)$ with a finite representation of the form

$$S=\sum_{j\in\mathbb{N}_m^n}\lambda_j\varphi_{j_1}^1\times\cdots\times\varphi_{j_n}^n b_j.$$

Hence, by Hölder's inequality and Definition 1.3 it follows that

$$\begin{aligned}|\Psi_T(S)|&\le\left\|(\lambda_j)_{j\in\mathbb{N}_m^n}\right\|_r\left\|\left(T\left(\varphi_{j_1}^1,\ldots,\varphi_{j_n}^n\right)(b_j)\right)_{j\in\mathbb{N}_m^n}\right\|_{r'}\\&\le\|T\|_{mas(r';r'_1,\ldots,r'_n;s')}\left\|(\lambda_j)_{j\in\mathbb{N}_m^n}\right\|_r\prod_{k=1}^n\left\|\left(\varphi_i^k\right)_{i=1}^m\right\|_{w,r'_k}\left\|(b_j)_{j\in\mathbb{N}_m^n}\right\|_{w,s'}.\end{aligned}$$

This shows that

$$|\Psi_T(S)| \leq \|T\|_{mas(r';r'_1,\ldots,r'_n;s')} \|S\|_{VN_f,(r;r_1,\ldots,r_n;s)},$$

for all $S \in \mathcal{L}_f(X_1,\ldots,X_n;Y)$.

Since on $\mathcal{L}_f(X_1,\ldots,X_n;Y)$, under our hypothesis for $X_1,\ldots,X_n$, we have

$$\|\cdot\|_{VN_f,(r;r_1,\ldots,r_n;s)} = \|\cdot\|_{VN,(r;r_1,\ldots,r_n;s)},$$

we conclude that Ψ_T is continuous on $\mathcal{L}_f(X_1,\ldots,X_n;Y)$ for $\|\cdot\|_{VN,(r;r_1,\ldots,r_n;s)}$ and

$$\|\Psi_T\| \leq \|T\|_{mas(r';r'_1,\ldots,r'_n;s')}.$$

By Proposition 2.3 (i), $\mathcal{L}_f(X_1,\ldots,X_n;Y)$ is dense in $\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)$. Hence we can extend Ψ_T to a continuous functional $\widetilde{\Psi}_T$ on $\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)$ in a unique way, with

$$\left\|\widetilde{\Psi}_T\right\| \leq \|T\|_{mas(r';r'_1,\ldots,r'_n;s')}.$$

Finally we note that $\mathcal{B}(\widetilde{\Psi}_T) = T$.

To show the reverse inequality let $\Psi \in \left(\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n;s)}(X_1,\ldots,X_n;Y)\right)^*$ and consider the corresponding n-linear mapping $\mathcal{B}(\Psi) \in \mathcal{L}(X_1^*,\ldots,X_n^*;Y^*)$, defined by $\mathcal{B}(\Psi)\left(\phi^1,\ldots,\phi^n\right)(b) = \Psi\left(\phi^1 \times \cdots \times \phi^n b\right)$, for $\phi^k \in X_k^*$, $k = 1,\ldots,n$ and $b \in Y$. Let us consider $n \in \mathbb{N}$ and $\varphi_{j_k}^k \in X_k^*$, for $k = 1,\ldots,n$, and $(b_j)_{j\in\mathbb{N}_m^n} \in l_{s'}^w(Y;\mathbb{N}_m^n)$. There is $(\lambda_j)_{j\in\mathbb{N}_m^n} \in l_r(\mathbb{N}_m^n)$ such that $\left\|(\lambda_j)_{j\in\mathbb{N}_m^n}\right\|_r = 1$ and

$$\left\|\left(\mathcal{B}(\Psi)\left(\varphi_{j_1}^1,\ldots,\varphi_{j_n}^n\right)(b_j)\right)_{j\in\mathbb{N}_m^n}\right\|_{r'} = \sum_{j\in\mathbb{N}_m^n} \lambda_j \left|\mathcal{B}(\Psi)\left(\varphi_{j_1}^1,\ldots,\varphi_{j_n}^n\right)(b_j)\right|$$

Now we can choose α_j, $|\alpha_j| = 1$, $j \in \mathbb{N}_m^n$ such that

$$\begin{aligned}\sum_{j\in\mathbb{N}_m^n} \lambda_j \left|\mathcal{B}(\Psi)\left(\varphi_{j_1}^1,\ldots,\varphi_{j_n}^n\right)(b_j)\right| &= \sum_{j\in\mathbb{N}_m^n} \lambda_j\alpha_j \mathcal{B}(\Psi)\left(\varphi_{j_1}^1,\ldots,\varphi_{j_n}^n\right)(b_j) \\ &= \Psi\left(\sum_{j\in\mathbb{N}_m^n} \lambda_j\alpha_j\varphi_{j_1}^1 \times \cdots \times \varphi_{j_n}^n b_j\right) = (*).\end{aligned}$$

By the continuity of Ψ and the Hölder's inequality we have

$$\begin{aligned}(*) &\leq \|\Psi\| \left\| \left(\lambda_{j_k}\alpha_{j_k}\right)_{j_k\in\mathbb{N}_m} \right\|_r \prod_{k=1}^{n} \left\| \left(\varphi^k_{j_k}\right)_{j\in\mathbb{N}^n_m} \right\|_{w,r'_k} \left\| \left(b_j\right)_{j\in\mathbb{N}^n_m} \right\|_{w,s'} \\ &= \|\Psi\| \prod_{k=1}^{n} \left\| \left(\varphi^k_{j_k}\right)_{j\in\mathbb{N}^n_m} \right\|_{w,r'_k} \left\| \left(b_j\right)_{j\in\mathbb{N}^n_m} \right\|_{w,s'} .\end{aligned}$$

This shows that $\mathcal{B}(\Psi) \in \mathcal{L}_{mas}^{(r';r'_1,\ldots,r'_n;s')}(X_1^*, \ldots, X_n^*; Y^*)$ and

$$\|\mathcal{B}(\Psi)\|_{mas(r';r'_1,\ldots,r'_n;s')} \leq \|\Psi\| .$$

∎

If we replace $\mathbb{N}^n$ by $\mathbb{N}$ and s' by ∞ in Theorem 3.4, we obtain the following known cases.

COROLLARY 3.5. *If the spaces X_k^*'s $(k = 1, \ldots, n)$ have the λ_k- bounded approximation property, then*

(i) *The topological dual of $\mathcal{N}_{(r;r_1,\ldots,r_n;s)}(X_1, \ldots, X_n; Y)$ is isometrically isomorphic to $\mathcal{L}_{as,(r';r'_1,\ldots,r'_n;s')}(X_1^*, \ldots, X_n^*; Y^*)$, for r, r_k and $s \in [1, +\infty]$, $k = 1, \ldots, n$ through the mapping $\mathcal{B}(\Psi)$ given as follows:*

$$\mathcal{B}(\Psi)\left(\phi^1, \ldots, \phi^n\right)(b) := \Psi\left(\phi^1 \times \cdots \times \phi^n b\right),$$

where Ψ is in the topological dual of $\mathcal{N}_{(r;r_1,\ldots,r_n;s)}(X_1, \ldots, X_n; Y)$, $\phi^k \in X_k^*$, $k = 1, \ldots, n$ and $b \in Y$.

(ii) *The topological dual of $\mathcal{L}_{VN}^{(r;r_1,\ldots,r_n)}(X_1, \ldots, X_n; Y)$ is isometrically isomorphic to $\mathcal{L}_{mas}^{(r';r'_1,\ldots,r'_n)}(X_1^*, \ldots, X_n^*; Y^*)$.*

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